analogy, it can be seen immediately that, if the insert has a finite angle at the re-entrant corner B, the stress will have a singularity at that point. Consequently, rounding these corners is imperative.

The symmetry of the cross section in Fig. 1 can be used to reduce the solution domain to a 45° sector, as shown in Fig. 3. The appropriate boundary conditions are also shown in this figure.

Conclusion

This problem is one of the simplest cases of antiplane elastic systems[4]; however, its solution appears to have been omitted from the literature on these systems. The reason for this apparent omission is not that it is difficult, but that it rarely occurs in applications. As was just shown, however, it is of great importance in the design of high-acceleration solid propellant rockets.

References

[1] Williams, M. L., Blatz, P. J., and Schapery R. A., "Fundamental studies relating to systems analysis of solid propellants," GALCIT 101, Guggenheim Astronaut. Lab., Calif. Inst. Tech. (February 1961).

[2] Bland, D. R., *The Theory of Linear Viscoelasticity* (Pergamon Press, New York, 1960), p. 87.

[3] Southwell, R. V., *Relaxation Methods in Theoretical Physics* (Oxford University Press, New York, 1946), p. 38.

[4] Milne-Thomson, L. M., *Antiplane Elastic Systems* (Academic Press, New York, 1962), Chap. 1.

Comments

Comments on "Dynamic Response of an Elastic Plate to a Moving Line Load"

William E. Thompson*
Cornell Aeronautical Laboratory Inc., Buffalo, N. Y.

In a recent investigation,[1] a problem very similar to that reported by Reismann[2] was studied. Although only a moderate effort was expended, it did extend over a considerable period of time so that most of the pertinent literature on the subject was ultimately identified. Without making an extensive list of references, two matters stand out. First, Mindlin[3] has stated that, except for constants, the equation of motion of the elastic plate strip (plate in plane strain) and the equation of motion of a simple beam are identical. Thus, the solutions for the dynamic response of a beam subject to a moving line load and supported in comparable ways become available for use on the corresponding plate problems. Second, in view of this, the work of Ludwig,[4] Kenney,[5] Mathews,[6] and Fryba[7] contains most of the essential results now reported anew by Reismann.

It is true that the work of Kenney and Mathews involves a linear restoring force term in the equation of motion due to the influence of a continuous elastic foundation, but a number of simplified, special cases among the several references allow one to anticipate Reismann's results.

It is regrettable that the author has not acknowledged this earlier work, especially in view of the recent, related paper by Lloyd and Miklowitz[8] in which both the references and the close association of the beam and plate problems were called to the reader's attention.

Received March 24, 1963; revision received July 12, 1963.
* Principal Mechanical Engineer. Member AIAA.

References

[1] Thompson, W. E., "Analysis of dynamic behavior of roads subject to longitudinally moving loads," Rept. VJ-1620-V-1, Cornell Aeronaut. Lab. Inc. (June 1962).

[2] Reismann, H., "Dynamic response of an elastic plate strip to a moving line load," AIAA J. **1**, 354–360 (1963).

[3] Mindlin, R. D., "Waves and vibrations in isotropic, elastic plates," *Proceedings, First Symposium on Naval Structural Mechanics* (Pergamon Press, New York, 1960), pp. 226–230.

[4] Ludwig, K., "Die Verformung eines beiderseits unbegrenzten elastisch gebetteten Geleises durch Lasten mit konstanter Horizontalgeschwindigkeit," *Proceedings, Fifth International Congress for Applied Mechanics* (John Wiley and Sons Inc., New York, 1939), pp. 650–655.

[5] Kenney, J. T., "Steady-state vibrations of beam on elastic foundation for moving loads," J. Appl. Mech. **21**, 359–364 (December 1954).

[6] Mathews, P. M., "Vibrations of a beam on elastic foundation," Z. Angew. Math. Mech., Part I, **38**, 105–115 (March–April 1958); Part II, **39**, 13–19 (January–February 1959).

[7] Fryba, L., "Schwingungen des Unendlichen, federnd, gebetteten Balkens unter der Wirking eines unrunden Rades," Z. Angew. Math. Mech. **40**, 170–184 (April 1960).

[8] Lloyd, J. R., and Miklowitz, J., "Wave propagation in an elastic beam or plate on an elastic foundation," J. Appl. Mech. **29E**, 459–464 (September 1962).

Reply by Author to W. E. Thompson

Herbert Reismann*
Martin Company, Denver, Colo.

The paper under discussion was taken from a more complete research report,[2] which gave a more detailed treatment and also discussed the connection of Ref. 1 with the subject of beam vibration. Appended to the research report[2] is a rather complete Bibliography pertaining to the beam problem. Since the paper dealt with the subject of plates and because of the usual space restriction, only references pertaining to plates were given in Ref. 1.

Thompson implies that the solution of the problem of the beam on an elastic foundation subject to moving load will yield the results of Ref. 1. Although there are certain similarities, there are, at least, two obvious facts that indicate major differences:

1) The static wave length of an infinite beam on an elastic foundation is finite, whereas that of a plate strip is infinite (in the long direction).

2) The plate has a denumerable infinity of critical speeds, whereas the beam has only one.

Moreover, the plate of Ref. 1 is not in a state of plane strain, and therefore Mindlin's analogy, referred to by Thompson, is not applicable. It should be pointed out that the results of Ludwig (Ref. 3, cited by Thompson) are in error for the case of supercritical speeds. Finally, it would have been impossible to acknowledge the work of Lloyd and Miklowitz (Ref. 5, cited by Thompson) inasmuch as it appeared in September 1962, whereas the paper under discussion[1] was submitted in August 1962.

References

[1] Reismann, H., "Dynamic response of an elastic plate strip to a moving line load," AIAA J. **1**, 354–360 (1963).

[2] Reismann, H., "Dynamic response of elastic plates to moving loads," Martin Co. Res. Rept. R-62-8 (July 1962).

Received May 15, 1963, revision received August 2, 1963.
* Chief, Solid Mechanics Section.